Original Article

ISSN: 2349 - 4891

International
Journal of Recent Research and Applied Studies
(Multidisciplinary Open Access Refereed e-Journal)

Dimensioning and Calibration of a mathematical model : Case of the Convection model, Constant coefficient diffusion and Dissipation and source term (Republic of Congo)

NGOUBOU Roch Corneille, DINGA Jean Bienvenu et NGANGA Dominique

[1]*Laboratory of Physico-chemical analysis of the Institute for Research in Exact and Natural Sciences, Scientific Research (IRSEN).*
[2]*Faculty of Science and Technology, Doctoral Training in Environmental Geosciences (Marien NGOUABI University).*

Received 25th September 2019, Accepted 1st October 2019

Abstract

This study focuses on the analysis of solutions of differential equations modeling a physical or chemical phenomenon ; This analysis is fundamentally based on a single solution, that of the resolution of the equation Convection, Diffusion and Dissipation at constant coefficient and source term by the method of decomposition of Adomian (Joseph BONAZEBI YINDOULA,2014). This analytical work has shown that the resolution of a differential equation or a differential equation system does not guarantee the reliability of the solution obtained, the latter will have to be validated in terms of :

- *Dimensioning ;*
- *Correlation with an experimental approach ;*
- *Validation of related data by statistical indices.*

The simulation by ground data of the solution of the equation of Convection, Diffusion and Dissipation at constant coefficient and source term resulting from the decomposition of Adomian has highlighted inadequacies of this solution : The existence of negative values on the variable (mass concentration) after digital application of ground data (GPS data) ; The solution resulting from Adomian's decomposition does not have the dimension of mass concentration and gives dimensional values : Dimensioning problem ; The solution resulting from the decomposition of Adomian has no parameters that characterize the pollutant that can be tracked : Calibration problem. This work is a significant contribution to improving the solution of the Convection, Diffusion and Dissipation equation at constant coefficient and source term derived from the decomposition of Adomian and thus proposes an algorithm of Dimensionand and Calibration of Adomian's solution.

Keywords: Mathematical Model, Equation to Dimensions, Transcendent Functions.

Introduction

This work aims to build an appropriate method of monitoring water pollutant by proceeding upstream by an analytical approach of some algorithms of transport of pollutant in water. The literature review identified the Constant and Term Coefficient Convection, Diffusion, and Dissipation equation as one of the pollutant transport equations in water; The analysis of the solution resulting from the Adomian decomposition method of this equation (Joseph BONAZEBI YINDOULA, 2014) has presented some insufficiencies which remove it from the experimentation: • the size of the solution; • The negativity of the values estimated by this solution; • No parameter in the expression of this solution characterizes the pollutant to follow; • The reliability of this solution. Our work consisted specifically in bringing a dimensioning and calibration to the ADM algorithm by the construction of two coefficients A and B having the dimension of a mass concentration in order to obtain an estimated approach of the measurements which is in phase with the experimentation.

Correspondence
NGOUBOU Roch Corneille
Corneillerochdesuz@gmail.com

Presentation of the equation of Convection, Diffusion and Dissipation at constant coefficient and term.

Model of Convection, Diffusion and Dissipation with constant coefficient and term. This model is formalized by:

$$\frac{\partial C(x,t)}{\partial t}+\frac{\partial C(x,t)}{\partial x}-\frac{\partial^2 C(x,t)}{\partial x^2}+C(x,t)=F(x,t) \quad (1)$$

Avec $F(x,t)=-A\mathrm{Sin}(x+t)+B$ and A, B are real numbers depending on pollutant concentration and position. Also, we consider that the transport of the pollutant in the water is done along the longitudinal axis.

Solution resulting from the Adomian Decomposition.

A solution of the model (1) by the decomposition of Adomian (J. Bonazebi, 2014) noted

ADM has for analytical expression:

$$\mathbf{C}(\mathbf{x},\mathbf{t}) = \mathbf{Cos}(\mathbf{x}) * \mathbf{Cos}(\mathbf{t})\ \mathbf{(2).}$$

With$(\mathbf{x},\mathbf{t}) \in [\mathbf{0};\mathbf{10}]\mathbf{x}[\mathbf{0};\mathbf{10}]$

Analysis of the Solution resulting from the Adomian Decomposition.

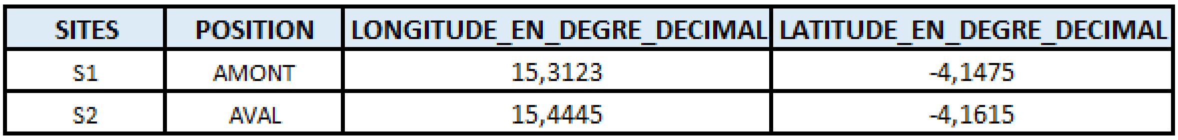
Land data

Location of Sampling points

The study is conducted on the Djiri River and two sampling points were selected based on their accessibility for sampling on the Djiri River (Table 5.1). The upstream (S1) and downstream (S2) points have geographic coordinates:

Table 1

Geographic Coordinates of Points



SITES	POSITION	LONGITUDE_EN_DEGRE_DECIMAL	LATITUDE_EN_DEGRE_DECIMAL
S1	AMONT	15,3123	-4,1475
S2	AVAL	15,4445	-4,1615

S1: Old Djiri Bridge 1st exit (Station)
S2: Former Djiri Bridge 2nd Exit

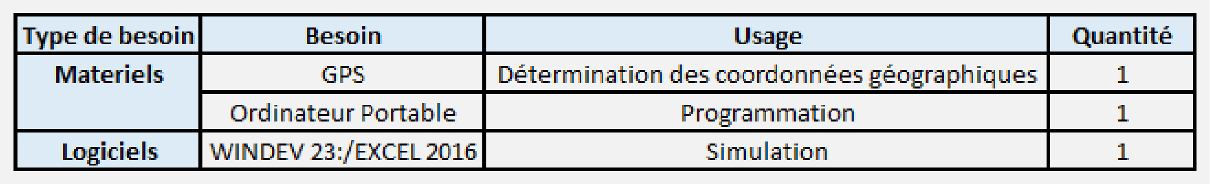
The upstream point (S1) is located 1km and the electric steel mill. All samples were taken over 12 months between May 2016 December 2017 in two points (upstream and downstream) chosen because of their accessibility and proximity to the target industrial site.

Materials

The material requirements and tools for collecting analytical samples, collecting input data and developing expected results are summarized in the table below:

Table 2

Materials and Tools



Type de besoin	Besoin	Usage	Quantité
Materiels	GPS	Détermination des coordonnées géographiques	1
	Ordinateur Portable	Programmation	1
Logiciels	WINDEV 23:/EXCEL 2016	Simulation	1

Results and Discussions

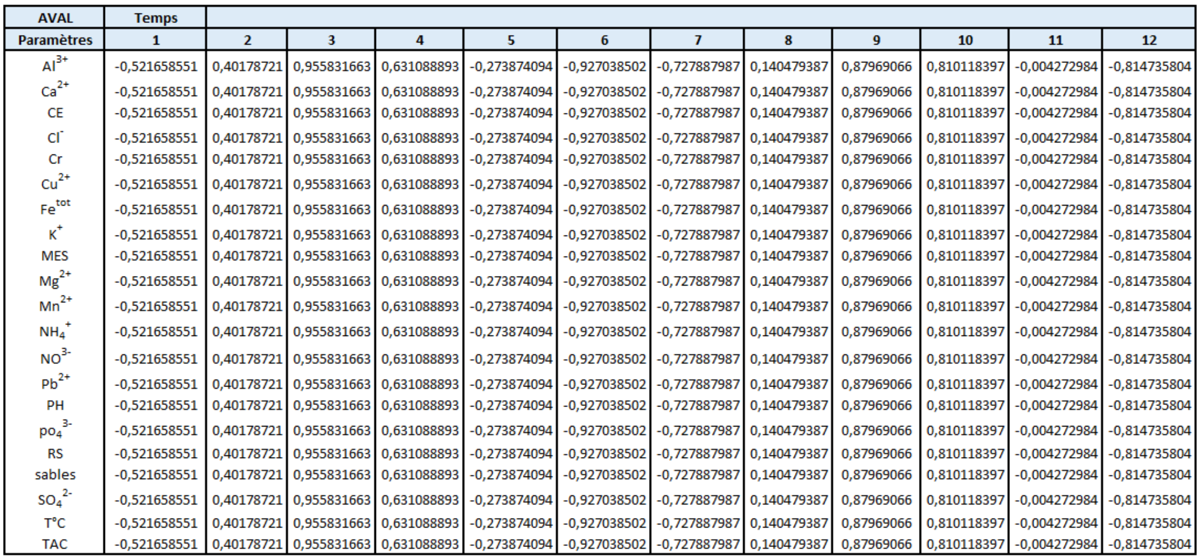
Results

Simulation of the ADM solution.

The numerical application of the expression (2) representative of the ADM solution gives the following results:

Table 3

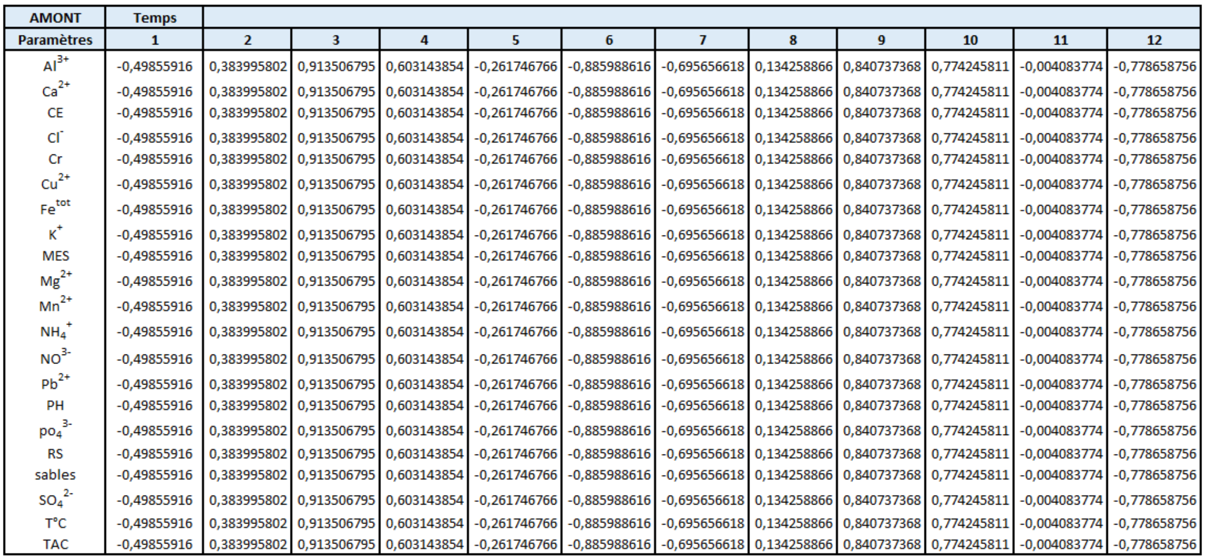
Downstream ADM Simulation



AVAL	Temps											
Paramètres	1	2	3	4	5	6	7	8	9	10	11	12
Al^{3+}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
Ca^{2+}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
CE	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
Cl^-	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
Cr	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
Cu^{2+}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
Fe^{tot}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
K^+	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
MES	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
Mg^{2+}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
Mn^{2+}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
NH_4^+	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
NO^{3-}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
Pb^{2+}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
PH	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
po_4^{3-}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
RS	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
sables	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
SO_4^{2-}	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
T°C	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804
TAC	-0,521658551	0,40178721	0,955831663	0,631088893	-0,273874094	-0,927038502	-0,727887987	0,140479387	0,87969066	0,810118397	-0,004272984	-0,814735804

Table 4

ADM Simulation at the Upstream Point



AMONT	Temps											
Paramètres	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Al^{3+}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
Ca^{2+}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
CE	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
Cl^-	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
Cr	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
Cu^{2+}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
Fe^{tot}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
K^+	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
MES	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
Mg^{2+}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
Mn^{2+}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
NH_4^+	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
NO^{3-}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
Pb^{2+}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
PH	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
po_4^{3-}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
RS	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
sables	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
SO_4^{2-}	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
T°C	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756
TAC	-0,49855916	0,383995802	0,913506795	0,603143854	-0,261746766	-0,885988616	-0,695656618	0,134258866	0,840737368	0,774245811	-0,004083774	-0,778658756

Comment

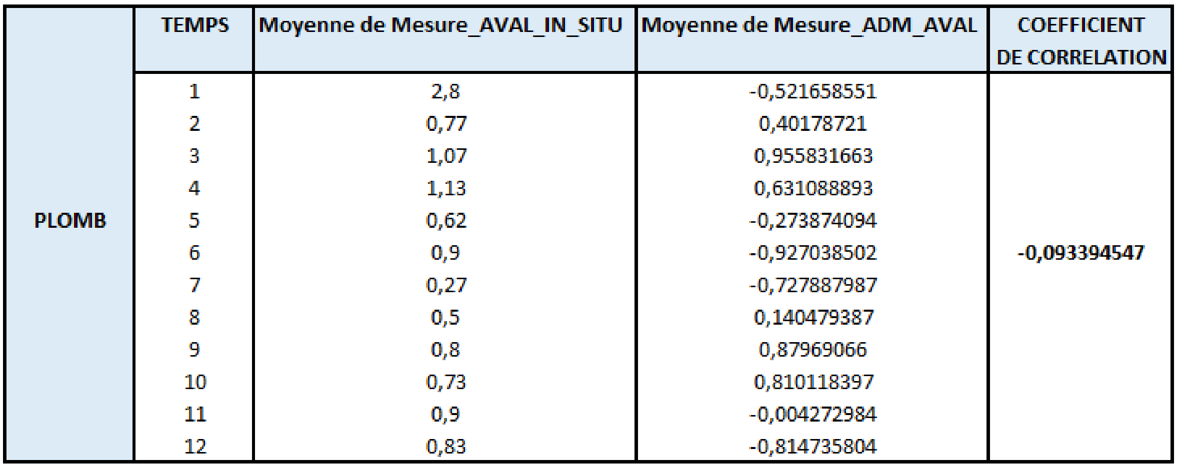
Tables 3 and 4 present the data estimated by the solution function of the equation of Convection, Diffusion and Dissipation at constant coefficient and source term. These two tables show that:

- The concentrations are identical each month for the parameters: the concentration does not depend on the physicochemical parameter;
- Existence of negative concentration values;
- Since the Cosinus mathematical function is transcendent, the estimated concentration value of the ADM solution is adimensional.

Correlative Analysis: Adomian Solution vs Experimental Approach.

Table 5

Determination Coefficient of Correlation in AVAL.



	TEMPS	Moyenne de Mesure_AVAL_IN_SITU	Moyenne de Mesure_ADM_AVAL	COEFFICIENT DE CORRELATION
PLOMB	1	2,8	-0,521658551	**-0,093394547**
	2	0,77	0,40178721	
	3	1,07	0,955831663	
	4	1,13	0,631088893	
	5	0,62	-0,273874094	
	6	0,9	-0,927038502	
	7	0,27	-0,727887987	
	8	0,5	0,140479387	
	9	0,8	0,87969066	
	10	0,73	0,810118397	
	11	0,9	-0,004272984	
	12	0,83	-0,814735804	

Table 6
Determination Coefficient of Correlation AMONT

	TEMPS	Moyenne de Mesure_AVAL_IN_SITU	Moyenne de Mesure_ADM_AVAL	COEFFICIENT DE CORRELATION
PLOMB	1	1,42	-0,49855916	**0,089031577**
	2	0,36	0,383995802	
	3	0,8	0,913506795	
	4	0,83	0,603143854	
	5	0,44	-0,261746766	
	6	0,5	-0,885988616	
	7	0,17	-0,695656618	
	8	0,3	0,134258866	
	9	0,7	0,840737368	
	10	0,6	0,774245811	
	11	0,5	-0,004083774	
	12	0,7	-0,778658756	

Comment

The analysis of these two tables shows correlation coefficients strictly correlation intervals [-1 ;-0.5] ou [0.5 ;1]: The correlation coefficient values 0.089 and 0.093 show that the estimated measurements (ADM) are moving away from the experimental measurements: No correlation between ADM MEXP.

Dimensioning and Calibration of the solution resulting from the Adomian Decomposition

- **Dimensioning and Calibration.**

The general form of our solution is :

$\mathbf{C(x,t) = ACos(x) * Cos(t) + B}$ (3).

Where A, B are calibration and sizing coefficients.
x: Longitudinal Cord in decimal degree at the point of sampling;
t: the sampling order;
The order is defined by the following function:

$\boldsymbol{\varphi : MXN^* \rightarrow N^*}$

$\mathbf{(m,n) \rightarrow n}$

$\varphi(m,n) = n$

$=$ sampling time or the ninth month of sampling

M= {1,2,3,4,5,6,7,8,9,10,11,12} ;

N^* : set of non $-$ zero natural numbers;

The determination of A and B results from the resolution of the linear system Next :

E. P=C where E is a square matrix of order 2 ;
E the component vector A and B ;
C thecomponent vectorC_1 et C_2.
C_1: Average measurement in situ upstream;
C_2:Average downstream in situ measurement;
This linear system is translated by:

$$\begin{bmatrix} \mathbf{Cos(X_{S1}) * Cos(t)} & \mathbf{1} \\ \mathbf{Cos(X_{S2}) * Cos(t)} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} = \begin{bmatrix} \mathbf{C_1} \\ \mathbf{C_2} \end{bmatrix}$$

The determinants associated with this linear system are:

$\Delta_{\text{Général}} = \mathbf{Cos(t) * (Cos(X_{S1}) - Cos(X_{S2}))}$

$\Delta_A = \mathbf{C_1 - C_2}$

$\Delta_B = \mathbf{Cos(t) * (C_2 * Cos(X_{S1}) - C_1 * Cos(X_{S2}))}$

The solutions of this linear system are given by:

$$\mathbf{A} = \frac{\Delta_A}{\Delta_{\text{Général}}} = \frac{\mathbf{C_1 - C_2}}{\mathbf{Cos(t)*(Cos(X_{S1}) - Cos(X_{S2}))}} \quad \textbf{(4).}$$

$$\mathbf{B} = \frac{\Delta_B}{\Delta_{\text{Général}}} = \frac{\mathbf{C_2*Cos(X_{S1}) - C_1*Cos(X_{S2})}}{\mathbf{Cos(X_{S1}) - Cos(X_{S2})}} \quad \textbf{(5).}$$

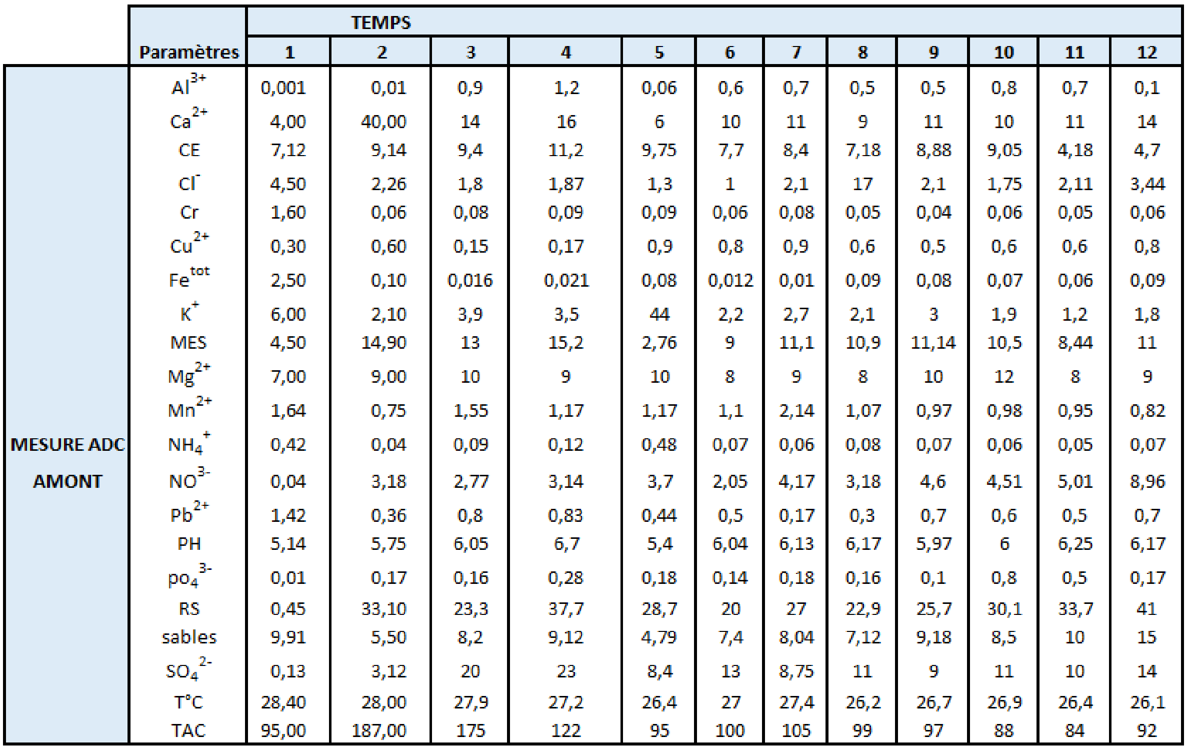
Coefficients A and B are specific to the pollutant monitored.

- **Simulation of the ADC solution**

The application of Algorithm III.1.6 to all the parameters evaluated gave the following results:

Table 7

Simulation of the ADC algorithm in Upstream

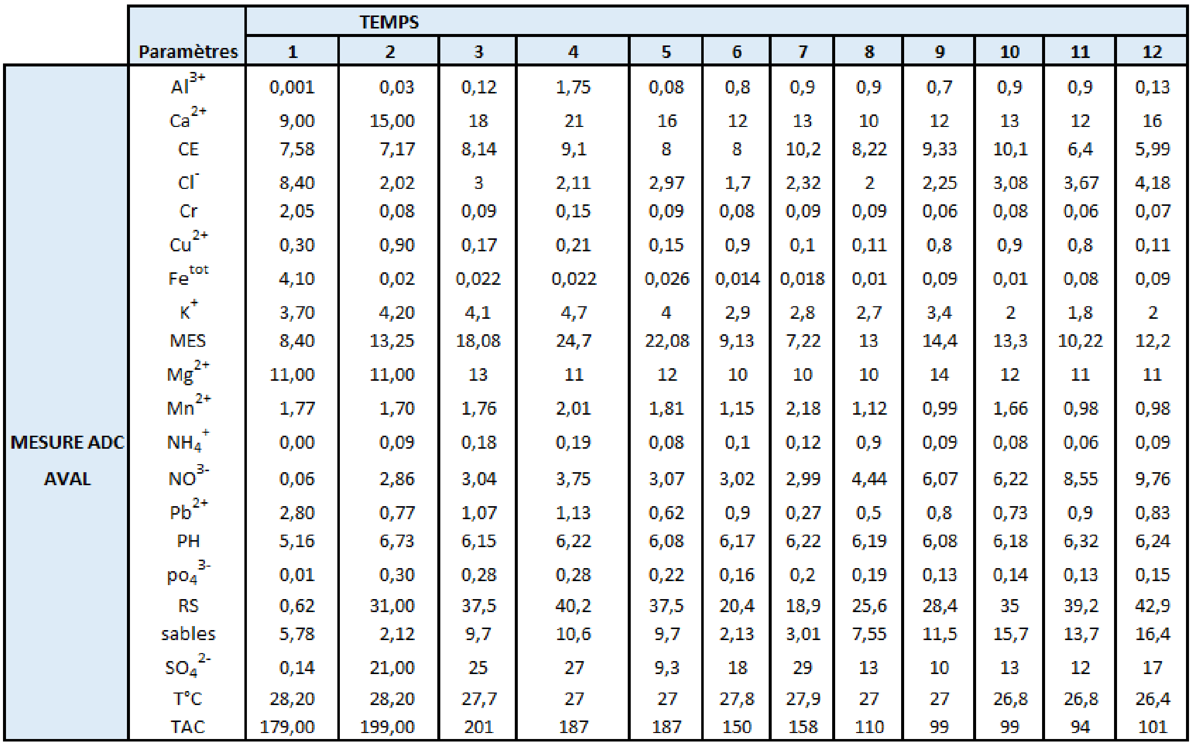


		TEMPS											
	Paramètres	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
MESURE ADC AMONT	Al^{3+}	0,001	0,01	0,9	1,2	0,06	0,6	0,7	0,5	0,5	0,8	0,7	0,1
	Ca^{2+}	4,00	40,00	14	16	6	10	11	9	11	10	11	14
	CE	7,12	9,14	9,4	11,2	9,75	7,7	8,4	7,18	8,88	9,05	4,18	4,7
	Cl^{-}	4,50	2,26	1,8	1,87	1,3	1	2,1	17	2,1	1,75	2,11	3,44
	Cr	1,60	0,06	0,08	0,09	0,09	0,06	0,08	0,05	0,04	0,06	0,05	0,06
	Cu^{2+}	0,30	0,60	0,15	0,17	0,9	0,8	0,9	0,6	0,5	0,6	0,6	0,8
	Fe^{tot}	2,50	0,10	0,016	0,021	0,08	0,012	0,01	0,09	0,08	0,07	0,06	0,09
	K^{+}	6,00	2,10	3,9	3,5	44	2,2	2,7	2,1	3	1,9	1,2	1,8
	MES	4,50	14,90	13	15,2	2,76	9	11,1	10,9	11,14	10,5	8,44	11
	Mg^{2+}	7,00	9,00	10	9	10	8	9	8	10	12	8	9
	Mn^{2+}	1,64	0,75	1,55	1,17	1,17	1,1	2,14	1,07	0,97	0,98	0,95	0,82
	NH_4^{+}	0,42	0,04	0,09	0,12	0,48	0,07	0,06	0,08	0,07	0,06	0,05	0,07
	NO^{3-}	0,04	3,18	2,77	3,14	3,7	2,05	4,17	3,18	4,6	4,51	5,01	8,96
	Pb^{2+}	1,42	0,36	0,8	0,83	0,44	0,5	0,17	0,3	0,7	0,6	0,5	0,7
	PH	5,14	5,75	6,05	6,7	5,4	6,04	6,13	6,17	5,97	6	6,25	6,17
	po_4^{3-}	0,01	0,17	0,16	0,28	0,18	0,14	0,18	0,16	0,1	0,8	0,5	0,17
	RS	0,45	33,10	23,3	37,7	28,7	20	27	22,9	25,7	30,1	33,7	41
	sables	9,91	5,50	8,2	9,12	4,79	7,4	8,04	7,12	9,18	8,5	10	15
	SO_4^{2-}	0,13	3,12	20	23	8,4	13	8,75	11	9	11	10	14
	T°C	28,40	28,00	27,9	27,2	26,4	27	27,4	26,2	26,7	26,9	26,4	26,1
	TAC	95,00	187,00	175	122	95	100	105	99	97	88	84	92

Source: Data estimated by the ADC algorithm.

Table 8

Simulation of the ADC algorithm in Aval



		TEMPS											
	Paramètres	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
MESURE ADC AVAL	Al^{3+}	0,001	0,03	0,12	1,75	0,08	0,8	0,9	0,9	0,7	0,9	0,9	0,13
	Ca^{2+}	9,00	15,00	18	21	16	12	13	10	12	13	12	16
	CE	7,58	7,17	8,14	9,1	8	8	10,2	8,22	9,33	10,1	6,4	5,99
	Cl^{-}	8,40	2,02	3	2,11	2,97	1,7	2,32	2	2,25	3,08	3,67	4,18
	Cr	2,05	0,08	0,09	0,15	0,09	0,08	0,09	0,09	0,06	0,08	0,06	0,07
	Cu^{2+}	0,30	0,90	0,17	0,21	0,15	0,9	0,1	0,11	0,8	0,9	0,8	0,11
	Fe^{tot}	4,10	0,02	0,022	0,022	0,026	0,014	0,018	0,01	0,09	0,01	0,08	0,09
	K^{+}	3,70	4,20	4,1	4,7	4	2,9	2,8	2,7	3,4	2	1,8	2
	MES	8,40	13,25	18,08	24,7	22,08	9,13	7,22	13	14,4	13,3	10,22	12,2
	Mg^{2+}	11,00	11,00	13	11	12	10	10	10	14	12	11	11
	Mn^{2+}	1,77	1,70	1,76	2,01	1,81	1,15	2,18	1,12	0,99	1,66	0,98	0,98
	NH_4^{+}	0,00	0,09	0,18	0,19	0,08	0,1	0,12	0,9	0,09	0,08	0,06	0,09
	NO^{3-}	0,06	2,86	3,04	3,75	3,07	3,02	2,99	4,44	6,07	6,22	8,55	9,76
	Pb^{2+}	2,80	0,77	1,07	1,13	0,62	0,9	0,27	0,5	0,8	0,73	0,9	0,83
	PH	5,16	6,73	6,15	6,22	6,08	6,17	6,22	6,19	6,08	6,18	6,32	6,24
	po_4^{3-}	0,01	0,30	0,28	0,28	0,22	0,16	0,2	0,19	0,13	0,14	0,13	0,15
	RS	0,62	31,00	37,5	40,2	37,5	20,4	18,9	25,6	28,4	35	39,2	42,9
	sables	5,78	2,12	9,7	10,6	9,7	2,13	3,01	7,55	11,5	15,7	13,7	16,4
	SO_4^{2-}	0,14	21,00	25	27	9,3	18	29	13	10	13	12	17
	T°C	28,20	28,20	27,7	27	27	27,8	27,9	27	27	26,8	26,8	26,4
	TAC	179,00	199,00	201	187	187	150	158	110	99	99	94	101

Source : Data estimated by the ADC algorithm.

Comment

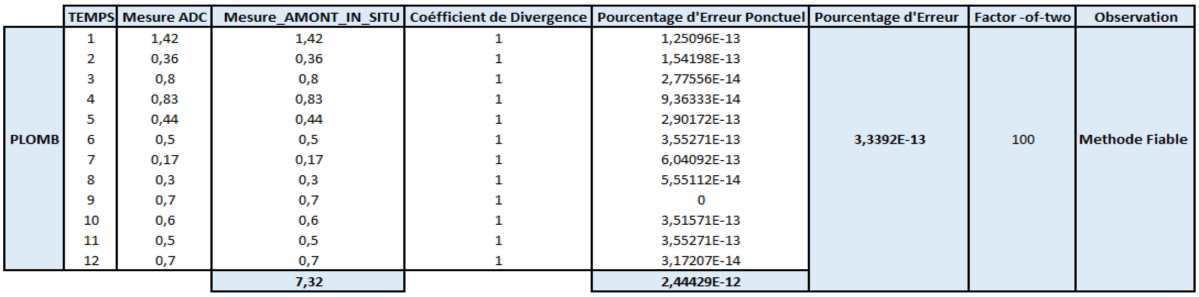
Tables 7 and 8 present the measurements estimated by the ADC algorithm.

• Validation of statistical indices.

Validation of the ADC solution is done through tests based on the statistical indices described in 3.1.5 and using the lead measurements for not taking the set of parameters; It results from the application of these indices the results inscribed in the tables:

Table 9

Determination of the Divergence Coefficient and Factor-of-two at the ADC vs. MEXP upstream point



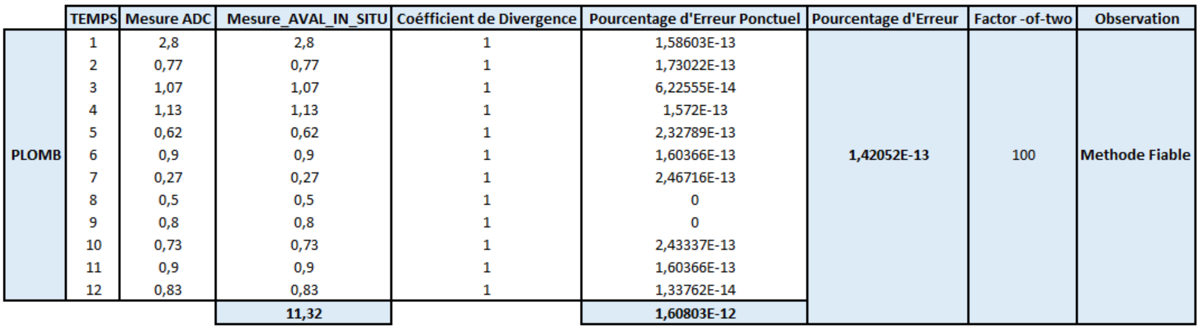
	TEMPS	Mesure ADC	Mesure_AMONT_IN_SITU	Coéfficient de Divergence	Pourcentage d'Erreur Ponctuel	Pourcentage d'Erreur	Factor -of-two	Observation
PLOMB	1	1,42	1,42	1	1,25096E-13	**3,3392E-13**	100	**Methode Fiable**
	2	0,36	0,36	1	1,54198E-13			
	3	0,8	0,8	1	2,77556E-14			
	4	0,83	0,83	1	9,36333E-14			
	5	0,44	0,44	1	2,90172E-13			
	6	0,5	0,5	1	3,55271E-13			
	7	0,17	0,17	1	6,04092E-13			
	8	0,3	0,3	1	5,55112E-14			
	9	0,7	0,7	1	0			
	10	0,6	0,6	1	3,51571E-13			
	11	0,5	0,5	1	3,55271E-13			
	12	0,7	0,7	1	3,17207E-14			
			7,32		**2,44429E-12**			

Comment

Table 9 shows a factor of two of 100%, a coefficient of divergence equal to 1 and thus reflects the reliability of the data estimated by the ADC approach, hence the reliability of this estimated upstream data approach.

Table 10

Determination of the Divergence Coefficient and Factor-of-two at ADC vs. MEXP upstream point with in situ measurements of lead.



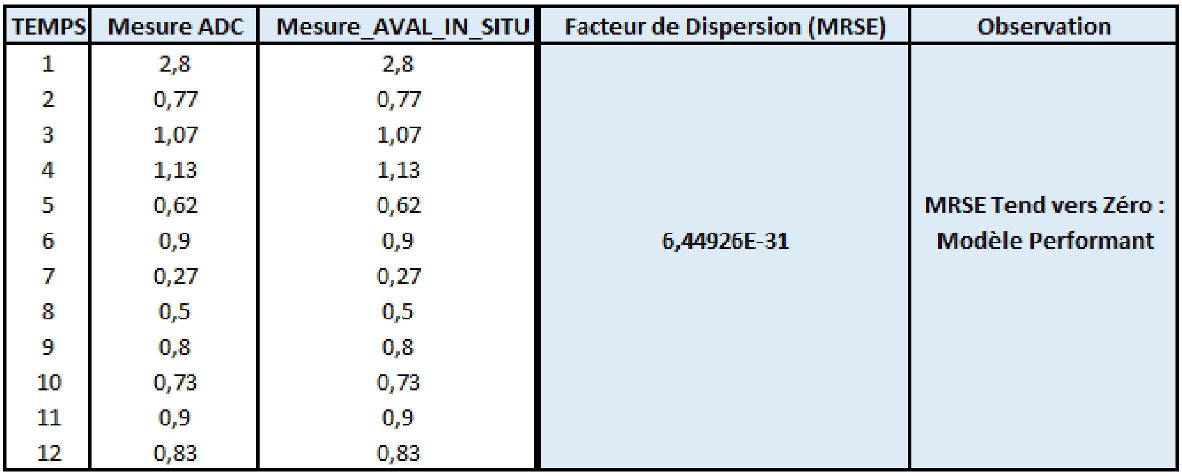
	TEMPS	Mesure ADC	Mesure_AVAL_IN_SITU	Coéfficient de Divergence	Pourcentage d'Erreur Ponctuel	Pourcentage d'Erreur	Factor -of-two	Observation
PLOMB	1	2,8	2,8	1	1,58603E-13	**1,42052E-13**	100	**Methode Fiable**
	2	0,77	0,77	1	1,73022E-13			
	3	1,07	1,07	1	6,22555E-14			
	4	1,13	1,13	1	1,572E-13			
	5	0,62	0,62	1	2,32789E-13			
	6	0,9	0,9	1	1,60366E-13			
	7	0,27	0,27	1	2,46716E-13			
	8	0,5	0,5	1	0			
	9	0,8	0,8	1	0			
	10	0,73	0,73	1	2,43337E-13			
	11	0,9	0,9	1	1,60366E-13			
	12	0,83	0,83	1	1,33762E-14			
			11,32		**1,60803E-12**			

Comment

Table 10 shows a factor of two of 100%, a coefficient of divergence equal to 1 and thus reflects the reliability of the data estimated by the ADC approach, hence the reliability of this estimated downstream data approach.

Table 11

Downstream dispersion factor between ADC and MEXP with in situ measurements of lead



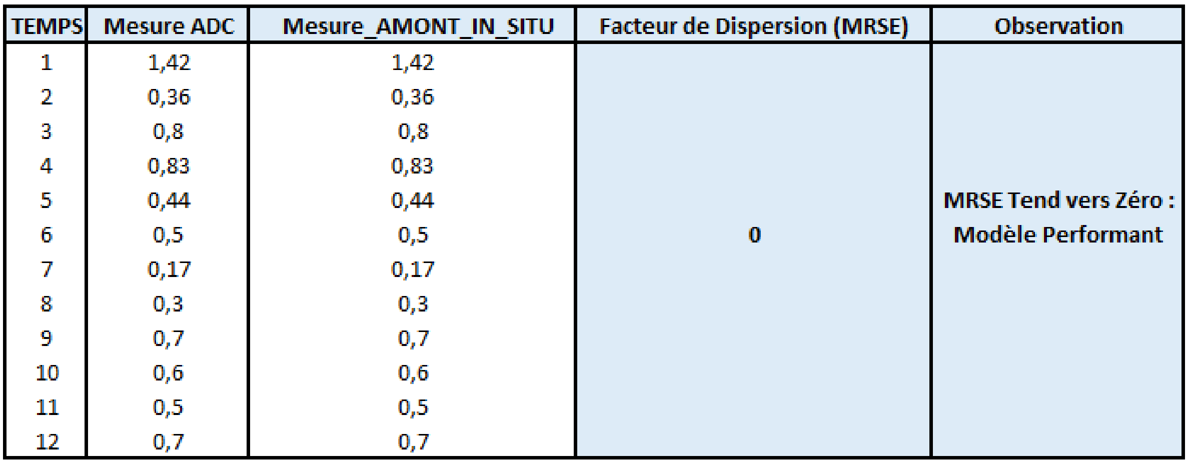
TEMPS	Mesure ADC	Mesure_AVAL_IN_SITU	Facteur de Dispersion (MRSE)	Observation
1	2,8	2,8	6,44926E-31	MRSE Tend vers Zéro : Modèle Performant
2	0,77	0,77		
3	1,07	1,07		
4	1,13	1,13		
5	0,62	0,62		
6	0,9	0,9		
7	0,27	0,27		
8	0,5	0,5		
9	0,8	0,8		
10	0,73	0,73		
11	0,9	0,9		
12	0,83	0,83		

Comment

Table 11 presents an MRSE that tends to zero and shows that the estimated approach of ADC data performs well downstream.

Table 12

Dispersion factor upstream between ADC and MEXP with in situ measurements of lead



TEMPS	Mesure ADC	Mesure_AMONT_IN_SITU	Facteur de Dispersion (MRSE)	Observation
1	1,42	1,42	0	MRSE Tend vers Zéro : Modèle Performant
2	0,36	0,36		
3	0,8	0,8		
4	0,83	0,83		
5	0,44	0,44		
6	0,5	0,5		
7	0,17	0,17		
8	0,3	0,3		
9	0,7	0,7		
10	0,6	0,6		
11	0,5	0,5		
12	0,7	0,7		

Comment

Table 12 presents an MRSE that tends to zero and shows that the estimated approach of ADC data performs well upstream.

- **ADC Dimensionality test**

This is to check the dimensional equation on the expression of the ADC solution.

The expression of the ADC solution being :

$\mathbf{F(X,T)_{A,B} = A * COS(X) * COS(T) + B}$

This expression is a parametric function having A and B for parameter. The transition to the equation to the dimensions at the point $\mathbf{X_1}$ **et temps t**gives:

$[\mathbf{F}(\boldsymbol{X_1},\mathbf{t})_{\mathbf{A,B}}]=[\mathbf{A} * \mathbf{COS}(\boldsymbol{X_1}) * \mathbf{COS}(\mathbf{t}) + \mathbf{B}]=\mathbf{[A+B]}$

$=[\frac{C_1-C_2}{Cos(t)*(Cos(X_{S1})-Cos(X_{S2}))} * \boldsymbol{COS}(\boldsymbol{X_1}) * \boldsymbol{COS}(\boldsymbol{t}) + \frac{C_2*Cos(X_{S1})-C_1*Cos(X_{S2})}{Cos(X_{S1})-Cos(X_{S2})}]$

$=[\mathbf{C_1 - C_2}]=\frac{[M]}{[V]}$because the cosinus function is transcendent and

COS(X_1) et COS(t)are dimensionless.

The verification at point $\mathbf{X_2}$ and at time t, will give the same conclusion.

- **Correlation ADC vs MEXP**

Table 13

Downstream Correlation MEXP vs ADC Test

	AVAL			COEFFICIENT DE CORRELATION
	TEMPS	MEXP	ADC	
PLOMB	1	2,8	2,8	1,000
	2	0,77	0,77	
	3	1,07	1,07	
	4	1,13	1,13	
	5	0,62	0,62	
	6	0,9	0,9	
	7	0,27	0,27	
	8	0,5	0,5	
	9	0,8	0,8	
	10	0,73	0,73	
	11	0,9	0,9	
	12	0,83	0,83	

Table 14

MEXP Upstream Correlation Test vs. ADC

	AMONT			COEFFICIENT DE CORRELATION
	TEMPS	MEXP	ADC	
PLOMB	1	1,42	1,42	1,000
	2	0,36	0,36	
	3	0,8	0,8	
	4	0,83	0,83	
	5	0,44	0,44	
	6	0,5	0,5	
	7	0,17	0,17	
	8	0,3	0,3	
	9	0,7	0,7	
	10	0,6	0,6	
	11	0,5	0,5	
	12	0,7	0,7	

Comment

Tables 13 and 14 show the correlation between the ADC solution and the experimental approach (MEXP).

Discussion

The set of results illustrated in 3.1 shows that:- As the transport of the pollutant in water is a physical phenomenon thus described by a variable of interest which is a dimensioned and defined positive physical quantity (Mass concentration) then the concentration of the pollutant in the water estimated by the ADM solution would be dimensioned and positive definite: which is absurd because the solution expression in the sense of Adomian of the equation of Convection, Diffusion and Dissipation with constant coefficient and term is adimensional (the cosine function is transcendental) and presents negative values; ADC solution is dimensioned and defined positive and conforms to the physical definition of a mass concentration; This solution is similar to that of the so-called ADE Advection equation (Dass JABBOUR, 2006), which also shows that the concentration of the pollutant in water at a point x and at time is estimated by a Gaussian-type expression given by:

$$\mathbf{C(x,t)} = \frac{\mathbf{M_1}}{\boldsymbol{\sigma_X\sqrt{2\pi}}} * \mathbf{e}^{-\frac{1}{2}\left(\frac{x}{\sigma_X}\right)^2} \ \textbf{\textit{o}}\text{ù}\ \boldsymbol{\sigma_X} = \boldsymbol{f(t)}.$$

The results of the analysis of the solution in the sense of Adomian of the equation of the equation of Convection, Diffusion and Dissipation with constant coefficient and term also challenges the mathematicians on the need to always carry out simulations of the models before publication or always to ensure that the approximations made during the resolution of the problem can not affect the quality of the expected result: Case of a physical phenomenon which in principle is analyzed from a variable of interest (physical quantity measurable experimentally) but estimated by an adimensional algorithm; Numerous statistical indices make it possible to evaluate the predictive performance of models or methods in relation to reality (Hanna et al., 1991, Mosca et al., 1998, Baléo et al., 2003); The statistical indices prove that the ADC solution is reliable since the Factor-of-two is 100% and efficient because the dispersion factor (MRSE) tends to zero.

Conclusion

This work has focused on the careful analysis of the solution according to the Adomian decomposition of the Convection Equation, Diffusion and Dissipation equation with constant and term coefficient (J. Bonazebi, 2014); this analysis revealed shortcomings on this solution and extended to the calibration and sizing of this solution. The ADC solution obtained from the calibration and dimensioning of the solution in the sense of Adomian is a contribution to the improvement of the solution in the sense of Adomian and will allow the monitoring of water pollutants.

Reference

1. Joseph Bonazebi Yindoula, Pare Youssouf, Gabriel Bissanga, Francis Bassano, Blaise Some, 2014 ; Application of the Adomian decomposition method and Laplace transform method to solving the convection Diffusion-Dissipation Equation.
2. Daas Jabbour, 2006 ; Étude expérimentale et modélisation de la dispersion en champ lointain suite à un rejet accidentel d’un polluant miscible dans un cours d’eau : Application à la gestion de crise. Ecologie, Environnement. Université de Provence-Aix-Marseille I.
3. Evaluation des performances de prédictions des modèlesou celles des méthodes par rapport à la réalité (Hanna et al. 1991 ; Mosca et al. 1998 ; Baléo et al. 2003).

Please cite this article as: NGOUBOU Roch Corneille, DINGA Jean Bienvenuet NGANGA Dominique (2019). Dimensioning and Calibration of a mathematical model : Case of the Convection model, Constant coefficient diffusion and Dissipation and source term (Republic of Congo). *International Journal of Recent Research and Applied Studies,* 6, 10(1), 1-9.